

THE COST OF INTERNAL VARIETY: A NON-LINEAR OPTIMIZATION MODEL

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1. Introduction

In the past decades, Design for Manufacturing and Assembly [Boothroyd and Dewhurst 2002] has significantly improved the product quality and profitability of many manufacturing companies. DFMA and “Design to Cost” [Michaels and Wood 1989] enable engineers to create product designs which can be manufactured at low cost. But despite the improvements in product quality, many companies are now realizing that applying manufacturing-oriented design methodologies to a single product is not efficient. If single products are optimized to be cost-effective in manufacturing, the production costs for the whole product family can be sub-optimized. On one hand, an optimization of the single product, adjusts it precisely to given customer requirements and prevents the product from over-dimensioning, but on the other hand – it increases design variety, and therefore drives to bigger costs in inventory, logistic, machines set-up, etc.

In this paper the economic aspects of the internal variety in product design and manufacturing are analyzed, and non-linear decision model for designing the cost-effective product family is given. The proposed calculation method enables product managers to estimate the cost of introducing variety into their product family. The practical applications of the method helped to optimize the internal product variety and thus, significantly reduce production costs, without jeopardizing the market coverage.

In Chapter 2, the general information about the Cost of Variety concept is given and related work is presented. Next, in Chapter 3, the numerical model for the cost of internal variety calculation is proposed and the optimization algorithm is explained. The practical application of the model is described in Chapter 4. Finally, the short summary is drawn in Chapter 5.

2. The related literature

In today’s highly competitive product markets, a product family must be adaptable enough to be easily customized, therefore it requires a sound product platform approach [Ishii 1995]. This area of research is well advanced in two general aspects [Martin 1996], [Siddique 2000], [Robertson and Ulrich 1998]:

1. Design for Variety – here the minimum component variety to satisfy maximum functional variety is the main focus
2. Common and Modular Process – here the diversity in manufacturing processes is minimized.

According to Anderson [1997], the “Cost of Variety” is the sum of all the costs of attempting to offer customers variety of products that are produced in inflexible factories. This includes the actual costs of customizing or configuring products, all the setup costs, the costs of excessive parts, procedures, and processes, etc. Agility Center [2004] distinguishes two types of variety: Internal Variety, which is experienced within the design, manufacturing and distribution operations and External Variety, which

is seen by customers. The aim is to reduce internal variety yet maximize external variety. More precisely, the Cost of Variety is defined as following:

- Inventory – raw materials, WIP, finished goods, administration, floor space,
- Setup – labor cost, machinery utilization, resource utilization, kitting.
- Model Changeovers – tooling/labor changeover costs, plant downtime.
- Materials – MRP administration, parts administration & internal distribution, procurement.
- Operations – tooling, dies and fixtures over minimum, delays caused by too many differences.
- Marketing – product line management, lost sales due to stock-outs, forecast errors.
- Quality – cost of defects.
- Service – excessive variety of parts, spare parts variety.
- Flexibility – cost of flexible manufacture, support services and information systems.

Another way of estimating the cost of variety is to compare a company's current operation budget to the idealistic case of producing a single product with no variety manufactured in the same volumes as current operations. The difference between current operations costs and the single product scenario is the cost of variety.

Nidamarthi [2003] noticed that total cost of product variety is sum of Fixed and Variable Costs, and profit is obtained by subtracting this cost from revenues. Fixed Costs of Variety (FCV) are costs independent to the extent of variety in a product family (i.e. costs that occur even if no products are sold). These are usually building rent, machine depreciation, etc. Variable Costs of Variety (VCV) depend on extent and volume of product variety. These are usually material costs, assembly work hours, etc.

Some practical product managers propose very simple approach, called the "Cost of new item". They assume, that an introduction on any new item into production, costs given amount of money (e.g. 50'000 \$/year) due to increased fixed costs of variety.

3. The Cost of Variety – an Optimization Model

The approaches for cost-of-variety estimations, presented in previous chapter, are easily understandable, however, the precise calculations for real-world, internal variety cases cannot be simply performed. Therefore, the novel approach for the cost of internal variety optimization is proposed in this paper. Generally, the calculation procedure is based on the assumption that, an optimal number of pieces per variant relates to minimal total production cost. And total production cost consists of direct and indirect costs of all variants produced. The direct costs vary with modifications of production volume. The main goal is to find the optimal production volume per internal variant, minimizing the total manufacturing costs.

3.1 The goal functions

The goal function, F , of the analysis is minimized production cost, C , when producing K variants of **the given product**

$$F = \min C = \sum_{k=1}^K c_k \quad (1)$$

where:

- c_k – is the cost of the production of k -th variant of the product,
- K – maximal number of variants (product sizes).

3.2 The decision variables

The decision variable is the optimal number of product pieces per variant, x_k .

3.3. The constraints

The constraints are the minimal required number of product pieces per variant, n_k . Naturally, the total production volume before and after optimization (summed for all variants), must be equal each other:

$$\sum_{k=1}^K X_k = \sum_{k=1}^K n_k \quad (2)$$

Additional assumption states, that bigger (mechanically stronger) product variant can take over the pieces of smaller product variant(s):

$$X_k \leq \sum_k n_k, \text{ for } k=1, \dots, K \quad (3)$$

3.4 The cost model

The applied cost model assumes that cost c_k of the production of k -th variant is the sum of direct product cost c_k^D multiplied by number of pieces produced x_k , and indirect product cost c_k^{ID} .

$$C_k = c_k^D \cdot X_k + c_k^{ID} \quad (4)$$

where:

- c_k^D – direct manufacturing cost of product piece
(is basically the sum of material and labor costs needed to produce one piece)
- c_k^{ID} – indirect production costs, which cannot be simply allocated to given product piece
(documentation, inventory, overhead, energy, etc.)

In the proposed cost model, the direct costs can be the functions of production volume, $c_k^D = f(x_k)$. For example, c_k^D can (and obviously should) decrease with increased number of pieces produced, x_k . Based on the analysis of historical data of real-world cases, it was also assumed that, there is a linear relation between product quantity and variable costs, expressed in double-logarithmic graph, Figure 1.

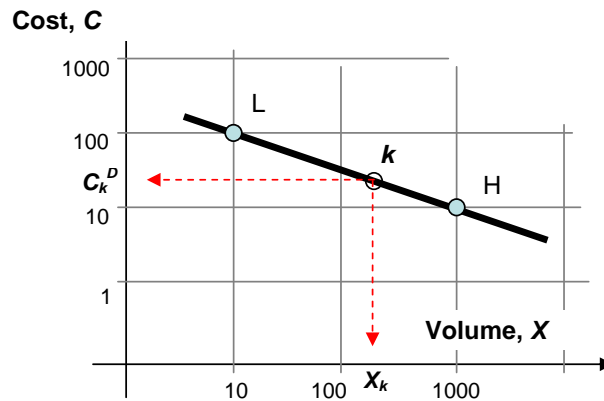


Figure 1. Exemplary relation between Cost and Product volume

Based on Figure 1, the actual manufacturing cost, c_k , related to given number of products, x_k , can be calculated as:

$$C_k = 10 \exp \left(\log C_L - (\log C_L - \log C_H) \frac{\log X_k - \log X_L}{\log X_H - \log X_L} \right) \quad (5)$$

where:

c_L – the manufacturing cost related to Low volume production, x_L

c_H – the manufacturing cost for High volume production, x_H

It is worth mentioning, that equation (5) can be applied to direct costs, c^D , mainly, while indirect cost component, c^{ID} , is constant for given variant, and exists if $x_k > 0$.

3.5 Model implementation

In summary, the optimization model minimizes the goal function describing the total cost of manufacturing, which is the sum of the production costs, generated by different variants manufacturing. The single variant manufacturing cost consists of direct and indirect cost components, which, in general, depend on variant production volume. The decision parameters are the number of products produced per given variant.

The mathematical model, described by equations (1) – (5) was implemented in MS/Excel environment, using specialized calculation modules: Analysis ToolPak and Solver. The optimization approach takes advantage of Generalized Reduced Gradient (GRG2) and Simplex algorithms.

The calculation procedure consists of a few main steps:

1. The minimal required number of pieces per variant must be specified at first.
2. Next, the direct and indirect cost components must be specified for both, short series (low volume production) and long series (high volume production).
3. Based on these data, the optimization algorithm modifies the number of pieces per variant, adjusts the piece costs (because of the new variant production volume), and calculates the total cost of production of all variants.
4. Step 3 is repeated, until the lowest total production cost is reached.
5. Finally, the results are given in two sections: (i) the optimal number of pieces per variant, and (ii) the total manufacturing cost.

4. The Practical Application

The proposed optimization algorithm has been successfully applied already to a few product families. In the following, the application of the approach to a family of spring mechanisms for HV apparatus is presented. Such as spring mechanisms power different types and sizes of HV apparatus, therefore should cover various energy levels (e.g. 17kJ, 15kJ, 12kJ, 9kJ). Increased product variety, adjusts and dimensions the spring mechanism modules to given performance levels, but on the other hand - it drives to bigger costs in inventory, logistic, machines set-up, etc. The goal of this analysis was to analyze the economic aspects of the variety in spring mechanism modules, and to propose the optimal set of product variants for cost-effective manufacturing.

As a starting point for analysis, four variants were originally planned, $K=4$, and the minimal required number of pieces per variant, n_k was determined (based on sales planes), Table 1.

Please note, that actual business figures have been edited to preserve confidentiality.

Table 1. Required number of pieces per variant

n_1 (17kJ)	n_2 (15kJ)	n_3 (12kJ)	n_4 (9kJ)	Σn_k
875	6'000	3'650	1'500	12'025

Following the equations (3) and (4), the constraints for optimal number of pieces per variant, x_k , can be described as shown in Table 2.

Table 2. Constraints for number of pieces per variant

x_1 (17kJ)	x_2 (15kJ)	x_3 (12kJ)	x_4 (9 kJ)	Σx_k
between: 875 and 12'025	between: 0 and 11'150	between: 0 and 5'150	between: 0 and 1'500	12'025

Next, based on the information received from part suppliers, the parameters of equation (5), namely: c_L, x_L, c_H, x_H , for all modules and main components of spring mechanism (discs, shafts, rods, levers, etc.) were estimated. The exemplary set of cost parameters for variants of an Opening Rod component is gathered in Table 3.

Table 3. Input data for an Opening Rod calculations

↓ Cost parameters		Variants →			
		$k=1$ (17kJ)	$k=2$ (15kJ)	$k=3$ (12kJ)	$k=4$ (9 kJ)
Nº of pieces for Low volume, x_L	#	875	6'000	3'650	1'500
Direct cost for Low volume, c_L	€	12.30	8.80	9.15	15.37
Nº of pieces for High volume, x_H	#	15'000	15'000	15'000	15'000
Direct cost for High volume, c_H	€	9.40	7.61	6.94	8.75
InDirect cost per Variant, c_k^{ID}	€	5'250	4'350	4'500	3'400

Subsequently, the optimisation algorithm was executed. Some of the results for an Opening Rod are shown in Table 4.

Table 4. The calculation results for an Opening Rod module

↓ Variant Configurations	Variants →				Total Mnfg. Cost [k€]
	$k=1$ (17kJ)	$k=2$ (15kJ)	$k=3$ (12kJ)	$k=4$ (9 kJ)	
	<i>number of pieces per variant, x_k</i>				
A (original)	875	6'000	3'650	1'500	132
B (cheapest)	875	11'150	-	-	109
C (single-variant)	12'025	-	-	-	120
D (most expensive)	6'875	-	3'650	1'500	134

Finally, when numerical calculations were performed for all modules of spring mechanism, the optimal arrangement of the product variants was found, Table 5. This table also shows the suggested number of variants per module.

Table 5. The summary of variant arrangements for spring mechanism

↓ Product modules	Variants →				No of variants
	$k=1$ (17kJ)	$k=2$ (15kJ)	$k=3$ (12kJ)	$k=4$ (9 kJ)	
Cam disk	17kJ	15kJ	15kJ	15kJ	2
Cam shaft	17kJ	15kJ	15kJ	15kJ	2
Fork	17kJ	17kJ	17kJ	17kJ	1
Operating lever	17kJ	15kJ	12kJ	12kJ	3
Opening rod	17kJ	15kJ	15kJ	15kJ	2
Operating lever shaft	17kJ	15kJ	12kJ	12kJ	3
...	

Table 5 shows the most cost-effective component size combinations for given variants. For example, the "Operating lever" should be produced in 3 sizes, optimised for 17kJ, 15kJ and 12kJ. In all product modules, the smallest energy level (9kJ) should be taken over by bigger sizes.

When analysing the total manufacturing costs of all components, it was found, that optimised product family is 21% cheaper compared to the original one.

5. Conclusions

The design for variety is a basic procedure supporting the managers and engineers to achieve the maximal market coverage. However, increased product variety increases manufacturing costs, what can negatively affect the profitability of the whole product family. In this paper, the method for analysis of the internal variety costs was proposed. The non-linear decision model, with the goal function minimizing the total manufacturing costs, and thus optimising the internal structure of product family was given and explained. The practical application of the method was described. It was also shown, that systematic analysis and optimisation of the internal variety costs helped to significantly decrease the total manufacturing costs. In presented case study the original number of product variants has been reduced from 4 to 2, what resulted in reduction of manufacturing cost about one – fifth.

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