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Device and software for educational simulation and design of dynamic dampers

Petri Makkonen, Huszár István,
Eric Coatanéa, Tanja Saarelainen
Helsinki University of Technology, TKK,
Department of Machine Design
Otakaari 4, PBOX 4100
02015 TKK
FINLAND
Petri.E.Makkonen@tkk.fi

Abstract

Free moving masses and cyclic free forces cause vibration problems in machine systems. The simplest way to reduce such vibrations is the minimization of freely moving masses and forces. In many cases this is not however possible. The addition of a dynamic damper to a machine system is a method to decrease vibrations in a critical component, and to transfer it to a passive mass. Design of such system must be done deliberately, since vibration response depends in a complex way of the geometry, stiffness and mass distribution of machine system. The paper describes a demonstration device with rotating free masses and a theory of small program for analysing the vibration characteristics of planar multi body spring mass system which can be used for engine vibration measurement test bench or for educational purpose.

Keywords: Linear System, Multi Body System, Vibration analysis, Dynamic damper

List of Symbols

A	nxn Adjacency matrix
A_{kl}	3x3 Adjacency submatrix connecting displacement of part l to elongation of spring k
C	n x n system damping matrix
I	n x n identity matrix
J_j	jth part's rotational inertia
K	n x n system stiffness matrix
M	n x n system mass matrix
N	Number of rigid bodies ground inc.
R	n x n system modal matrix
$\bar{f}$	n x 1 force excitation vector
i	index in system's d.o.f.
j	index of part, ground's $j = 0$
$k_{x,p}$	bushing p's elasticity in x-direction
$k_{y,p}$	bushing p's elasticity in y-direction
Δk_i	spring k's position in part i's frame
Δk_x	spring elements's x-position to global reference
Δk_y	spring elements's y-position to gl.f.
m_j	jth part's mass
n	system's degree of freedom
t	time
Δx_i	displacement of part i in i's frame
$\bar{x}$	system's displacement vector (in parts' frame)
w_i	relative excitation frequency
α	Rayleigh's mass-relative damp ratio
β	Rayleigh's stiffness-relative damp ratio
γ	Rigid body tilt angle
ζ_i	ith relative damping factor
$\zeta_{i,kr}$	ith critical damping factor
θ	ith phase
ϕ_i	ith eigen mode vector
ω	the frequency, which the system is exitated
ω_i	system's ith eigenfrequency

1. Introduction

Free rotating masses and other cyclic free forces cause vibrations in machine systems. For example, in reciprocating engines, significant mass forces, which are difficult to balance, can exist depending of the piston order configuration. Also in other machines, always, when a dominating, continuous, and sinus form free force exist, the designer is to face between choices: 1) to reduce the vibrating free mass 2) to increase the machine system's mass so that total vibration amplitude decreases 3) to reduce the systems vibration by viscous damping between frame and ground 4) to use dynamic dampers. Common to each of the first three choices are, that vibration can be reduced in a frame, while not totally eliminated. With dynamic dampers, vibration can be significantly reduced, but they operate only in a narrow region of operation conditions, and are difficult to design to fulfill their function.

The objective of this research has been development of simple demonstration device for students to motivate and to help to understand vibration problems in machine systems caused by free forces. The goal was to design and build a simple vibration system prototype, which can demonstrate the vibrations of a rigid, flexibly to ground mounted machine frame. The novelty of the system is easily reconfigurable excitation method, which is based on two rotating masses with syncronized use. This enables various sinus-formed vibrations simulating the vibrations of reciprocating engines. The unwanted vibrations can then be damped out by dynamic damper. The system was designed to be operated by two stepper motors, which enable various types of controlled excitations in any directions, and simple interfacing to a controlling computer, which in this case is a standard PC with one free parallel port.

2. Literature

Teaching of vibration problems has been traditionally quite math oriented and not in a focus of intensive pedagogical or didactic research. The teaching of mechanical systems vibration problems begins from learning and understanding one degree of freedom system's vibration. Various downloadable software and teaching material exist, and also web based sites, like one described in [1]. To understand single degree of freedom systems is a foundation, which can easily be generalised to linear N-degree of freedom systems. Rayleigh damping, enables easily understandable generalisation from forced and damped system of N-degree to be converted to N scalar single degree systems. There exists also teaching software [2], which goes directly to non-Rayleigh and non-harmonic forced systems, without going to the principles of coordinate transform, which is the essence of Rayleigh's method. A practical obstacle of forming the system of equations is the care taking of geometry; to learn the principles of free body system is fundamental. The general relation between geometry and displacements and elasticity is the essence of Finite Element Method and multi body systems analysis, which this paper also deals with. In industrial problems, most real world vibration analysis problems do not have a closed form mathematical solution, and non-linear analysis or other advanced methods are used. This leads to teaching with FEM-codes like ANSYS, ALGOR or NASTRAN, [3], which often are used as black box, usually giving for the user the behaviour of the planned machine system, without solid understanding, how it relates to the defining characteristics of design, the parameter dependency.

A research using multimedia courseware for teaching dynamic concepts of vibration phenomena [6] was conducted. The learning was assessed by pre and post test evaluation of three student groups' skill levels. The assessment material was from standard vibration texts, which were matched to the multimedia modules. The assessment instruments were geared towards to evaluate which concepts did the students learn and which not. The students received pre- and post-testing where the conceptual understanding was evaluated by asking them to: 1) Define concepts covered by the module 2) Discuss of their importance 3) To provide examples when the concepts are used. In general, the use of multimedia modules showed increase in learning. The multimedia modules helped students to understand concepts qualitatively and increased conceptual understanding and it's importance in engineering. However, it didn't help students to generate examples when the examples come into play. In [7], a experimental teaching device was developed, which is based on a lateral 1-DOF and 3-DOF system, which can be studied with impact hammer method. The focus is on mode shape identification and damping is not considered. The eigen frequencies are computed with Matlab and a the system is modelled also with ANSYS. The paper presents the theory and results of single analysis, however no impact on education was considered. Other works for educational multi degree freedom models or educational test rigs have been created. [8] present an experimental test rig setup, which enables identification of modal frequencies with an Finite Element Software from impact hammer measurements of a clamped free bar, and to perform a frequency analysis based on measurements. In [9], a test rig of one cylinder engine vibration measurement was developed, which identifies from LVD-transducers the vibrations response to a single degree of freedom model. The identification procedure was left to be presented in a later paper.

3. Methodology

3.1 Vibrations of N-degree system

When a linear spring-damper system with N-degrees of freedom is free and undamped, the governing dynamic equations are [4]:

$$M\ddot{\bar{x}} + K\bar{x} = 0 \tag{1}$$

By substituting

$$\begin{aligned} \bar{x} &= \bar{f}\cos\omega t \\ \ddot{\bar{x}} &= -\omega^2 \bar{f}\cos\omega t \end{aligned} \tag{2}$$

and by multiplying (1) with the inverse of mass matrix, M^{-1}, we get:

$$\left[M^{-1}M\omega^2 - M^{-1}K\right]\bar{f}\cos\omega t = 0 \tag{3}$$

which substitutes to a linear homogenous system

$$\left[M^{-1}K - \omega^2 I\right]\bar{x} = 0 \tag{4}$$

The eigen frequencies ω_i are solved from condition (4)

$$\det\left[M^{-1}K - \omega^2 I\right] = 0 \tag{5}$$

by substituting to (5)one by one we get the eigen vectors $\bar{\phi}_i$, for each eigen frequency ω_i:

$$\det\left[M^{-1}K - \omega_i^2 I\right]\bar{\phi}_i = 0, i = 1,..., N \tag{6}$$

To derive the equations for forced and damped case, there exists time dependent oscillating force $\bar{f}(t) = \bar{f}\cos\omega t$, and also damping C, thus the governing equation is:

$$M\ddot{\bar{x}} + C\dot{\bar{x}} + K\bar{x} = \bar{f}\cos\omega t \tag{7}$$

For simplicity, Rayleigh damping is assumed, thus

$$C = \alpha M + \beta K. \tag{8}$$

Now, because Rayleigh damping is assumed, the system (7) can be developed to a uncoupled system if new coordinate system called main coordinate system is used:

$$\bar{x} = R\bar{q} = \left[\bar{\phi}_1\bar{\phi}_2 \ldots \bar{\phi}_N\right]\bar{q} \tag{9}$$

When (8) is place to equation (7) and the result is multiplied with R^T we get

$$R^T MR\ddot{\bar{q}} + R^T CR\dot{\bar{q}} + R^T KR\bar{q} = R^T \bar{f}\cos\omega t \tag{10}$$

The system will now reduce to a uncoupled system of N scalar equations

$$\hat{M}_{ii}\ddot{\bar{q}}_i + \hat{C}_{ii}\dot{\bar{q}}_i + \hat{K}_{ii}\bar{q}_i = \bar{\phi}_i^T \bar{f}\cos\omega t \tag{11}$$

where new band matrices are defined

$$\begin{aligned} \hat{M} &= R^T MR \\ \hat{C} &= R^T CR \\ \hat{K} &= R^T KR \end{aligned} \tag{12}$$

From classic second order scalar equation damping and phase shift parameters can be solved:

$$\zeta_{cr,i} = 2\sqrt{\hat{K}_{ii}\hat{M}_{ii}} \tag{13}$$

$$\zeta_i = \hat{C}_{ii} / \zeta_{cr,i} \tag{14}$$

$$\tan\theta_i = \frac{2\zeta_i}{1 - w_i^2} \tag{15}$$

where w_i is relative frequency,

$$w_i = \omega / \omega_i \tag{16}$$

The response will be

$$\bar{x}(t) = R^T \bar{f}\cos\omega t \tag{17}$$

Any periodic excitation can be described in Fourier terms, and thus any periodic excitation will change the right hand side of (7) to form:

$$M\ddot{\bar{x}} + C\dot{\bar{x}} + K\bar{x} = \bar{f}_0 + \sum_{i=1}^{\infty} \bar{f}_{nc}\cos i\omega t + \bar{f}_{ns}\sin i\omega t \tag{18}$$

The vibration response is computed by superpositioning steps (9)-(14) for each Fourier term. The constant force response will be only static displacement from static balance point:

$$\bar{x}_0 = K^{-1}\bar{f}_0 \tag{19}$$

Total response will be

$$\bar{x}(t) = \bar{x}_0(t) + \sum_{i=1}^{\infty} \bar{x}_n(t) = \bar{x}_0 + \sum^{\infty} \bar{x}_n e^{in\omega t} \tag{20}$$

If the basic frequency ω or some of it's integer multiples is the same as some of the eigen frequencies, then the vibration will be strong and in same eigen mode as that eigen frequency.

3.2 Modelling the multi body system

The N-body system described in chapter 3.1 is programmed as solver core. A pre-processor was also implemented, forming the planar multi body system to N-degree of freedom system of form (7). First, the coordinate system of the planar multi body system is defined. The coordinate system for each body i has three coordinates x_i, y_i and γ_i, which describe the position and orientation of body with respect to the global (ground) origin. For simplification of the motion equations, initial rotational displacement for every body is assumed to be zero. The rotational vibration is assumed to be linear, thus $\gamma < 0.1$ radians. The motion equations are derived with the principle of Newton-Euler for each part separately from free body picture and thus mass matrix of the system is diagonal and the coefficients are:

$$M_{3j+1,\,3j+1} = m_j;\ \ M_{3j+2,\,3j+2} = m_j;\ \ M_{3j+3,\,3j+3} = J_j \tag{21}$$

The relation between un-displaced rigid body and spring position is:

$$\bar{x}_{k,i,org} = \bar{x}_i + R^{-1}(\gamma_i)\Delta\bar{k} \tag{22}$$

where

$$R(\gamma_i) = \begin{bmatrix} \cos(\gamma_i) & \sin(\gamma_i) \\ -\sin(\gamma_i) & \cos(\gamma_i) \end{bmatrix} \tag{23}$$

for the displaced rigid body yields:

$$\bar{x}_{k,i,org} = \bar{x}_i + R^{-1}(\gamma_i)\Delta\bar{x} + R^{-1}(\gamma_i)R^{-1}(\Delta\gamma_i)\Delta\bar{k} \tag{24}$$

For the displacement of rigid body then yield by subtracting (22) from (24):

$$\Delta\bar{x}_{k,i,org} = R^{-1}(\gamma_i)\left[\Delta\bar{x} + \left[R^{-1}(\Delta\gamma_i) - I\right]\Delta\bar{k}\right] \tag{25}$$

By little derivation we get the relation between rigid body displacement and spring elongation

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_\gamma \end{bmatrix} = A_{ik}\Delta\bar{x}_{ik} = \begin{bmatrix} \cos(\gamma_i) & -\sin(\gamma_i) & \Delta k_y \\ \sin(\gamma_i) & \cos(\gamma_i) & \Delta k_x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta\gamma_i \end{bmatrix} \tag{26}$$

The adjacency matrix describes the relation between all elongations of springs and all displacements of rigid bodies. It is roughly of form (27), where the row k describes the effect on spring k's elongation by displacement of the body l. (It depends on spring coupling between the rigid bodies, and submatrix A_{ki} describes the spring k's i part's effect on spring k while A_{kj} describes j parts effect on spring k.)

$$A = \begin{bmatrix} -A_{1i} & A_{1j} & \dots \\ -A_{2i} & A_{2j} & \dots \\ \dots & \dots & \dots \end{bmatrix} \qquad (27)$$

The accurate forming of adjacency matrix A is given in [5]. The relation between elongations and forces in springs is a band matrix, from which the total system band stiffness G matrix is formed:

$$\begin{bmatrix} F_x \\ F_y \\ M_\gamma \end{bmatrix}_k = G_{kk} \Delta \bar{\varepsilon}_k = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_\gamma \end{bmatrix}_{kk} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_\gamma \end{bmatrix}_k$$

The final system stiffness matrix K for system (7) is obtained by multiplication [5]:

$$K = A^T G A \qquad (28)$$

The C matrix is obtained for (7) from the Rayleigh coefficients of (8). The force vector is defined currently only in rigid body coordinate system, thus $\bar{f}$ is defined as it is in (7).

4. Results

4.1 Demo device

A simple demonstration device with six degree of freedom (DOF) containing two eccentric masses running opposite circles driven by stepper motors with computer control was designed and manufactured, Fig 1. The stepper motors were controlled with Bebek Electronic Inc. bipolar stepper motor drive M106 with a M108 multiplexer, capable of driving simultaneously four motors. In the multiplexer interface, each stepper motor is controlled by two lines (one for each coil) enabling total four motors for Centronix eight parallel lines bus. The stepping is made completely by computer with a separate program supplied by Bebek, even its very simple to create own customized driver programs in future. The syncronized control of two opposite rotating masses were implemented with drive definition file, which gradually performed a speed profile for two motors keeping the simultaneous syncronized opposite phase angle between motors enabling thus sinus formed common free mass vibration. The body was hanged with springs in all three translational directions enabling also all three rotational displacements. The syncronized stepper motor control of two motors enables creation of several types of excitations, but it is mainly used for pure vertical (Z-axis) sinus form excitation. The system was completed with dynamic mass damper capable to damp vertical vibration in Z-axis. The mass damper was designed separately. It was implemented simply from a dummy mass connected with ordinary compression springs against the surface of part with unwanted vibration, and it's dimensioning principles are very simple: It's mass is designed to be at least 4-5 % of the vibrating system's total mass and it's spring constant stiffness k is adjusted so, that it's own critical frequency is tuned to same as the unwanted excitation frequency. It has no viscous damping, and thus it wakes up very easily from unwanted surface’s vibration excitation, and decreases it significantly.

4.2 Computer program

A Matlab-program was written, which analyses the frequency response of a planar multi body system (DOF=3(N-1)) of any body (N) and spring number complexity. The software can be used to compute eigenfrequencies, -modes, and frequency response in position, but can be easily modified to analyze also speed and acceleration response. The model is configured as a text file, and solved by an analysis program with multi body pre-processor. A simple model graphics output is also supported, fig 3.

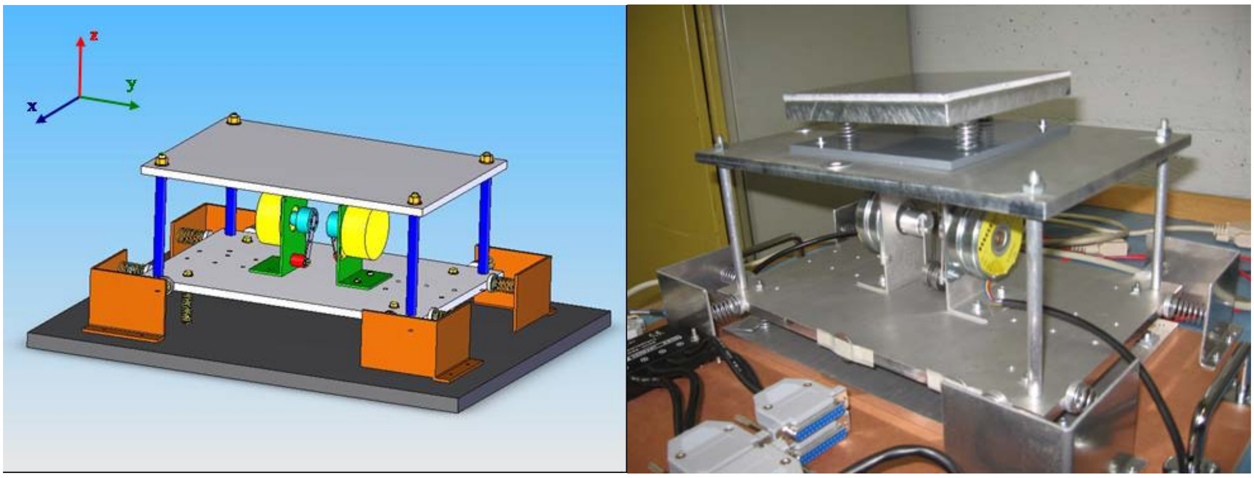


Figure 1. Model of original vibration block device (left) and implementation with damper (right).

4.3 Analysis results

The original block system in Fig 1 (left) was designed to have a significant translational response in Z-direction on a midrange of motor speed. The dimensions of the springs, and the vibration block other dimensions were designed with a separate Excel-program, not to have multiple eigen modes on the same frequency, fig 4. Initially the system was designed and tuned up as completely symmetrical on each DOF, thus the mass and stiffness matricies were completely band matricies. The tuning was made by impact hammer on each DOF separately. The Excel-program can compute roughly from spring coil data and the systems physical block dimensions, the eigen frequency of each DOF with multiple single-DOF model. The initially dimensioned springs had adequate compliance with final impact hammer measurements. The critical frequency was tested by sinus excitation and by impulse mode analysis, fig 2 (left). This was compared to the computed block model variant's response, fig 2 (right). In the second phase, a dynamical damper, fig 1 (right), was added to the system, which was designed to damp completely critical frequency of fig 2. The damped system's response is then shown in fig 3 (right).

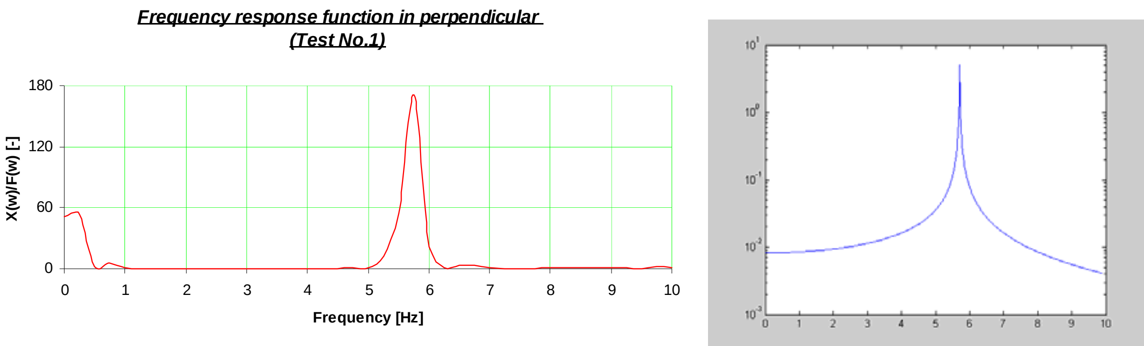
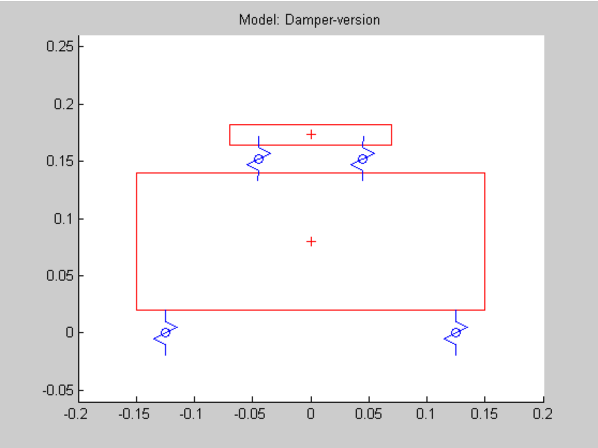


Figure 2. Measured (left) and computed (right) response of original vibration block device.

4.4 Design principles and test regimes

The system was mechanically designed to create and damp one dominating sinus formed excitation at time on any XYZ-direction and XY-position on the vibrating block by moving the motors. Thus, by running sweeps, all frequencies can be exited with the two stepper motor design. The test regime and operation range was designed to be between 0-10 Hz, and all of the six critical frequencies of the vibrating block were designed to be at that range, to be able to demonstrate pedagogically all six eigen modes, however, no multiple frequencies were allowed. A typical demonstration test is a Z-directional sinus-sweep, which as a vertical positional response, shown simulated in figure 3 (right). Vibration data aquisition has not yet been implemented, this is discussed in chapter 5, Discussion.



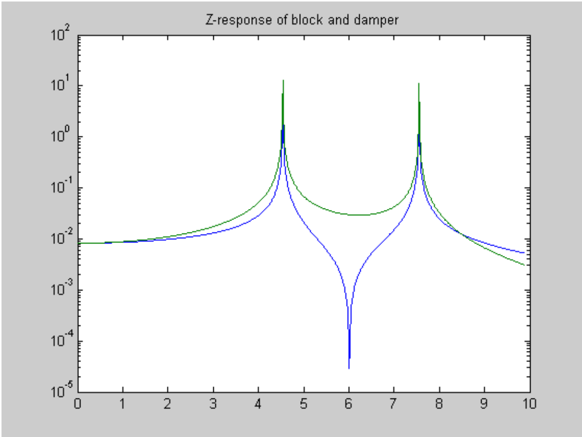


Figure 3. Computer program visualisation of the damper model (left) and it's frequency response of frame and damper (right).

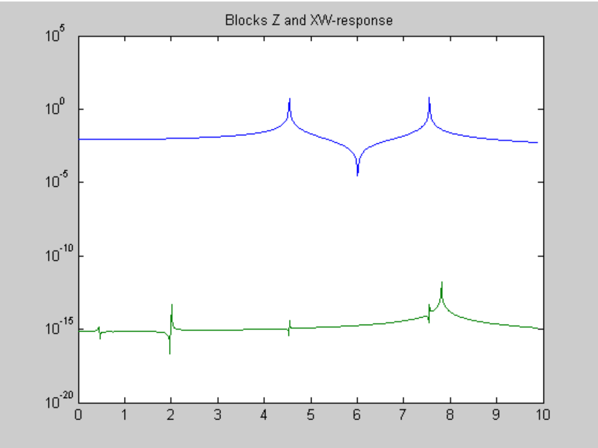


Figure 4. Rotational and vertical response of damper version's block vibrations.

5. Summary and discussion

5.1 Summary

The paper presents a device capable to demonstrating vibration problems and a method to overcome it when specific requirements are fulfilled, with a dynamic damper. The solving of such problems requires understanding of multi body vibration dynamics fundaments. To learn the basics, a simple program was created based on Rayleigh damping. This program analyses in frequency plane the frequency response of N-degree of freedom system. To help the creation of the system equations, a automatic pre-processor for planar linear multi body systems conversion to linear system of equations was created. The theoretic background is briefly documented in this paper, and together with the device, it's easy to teach and demonstrate design of dynamic dampers and management of vibration problems.

5.2 Discussion

When compared to reality, the restriction to planarity is pedagogical, since the geometry is easy to implement with graphical pre-processor, and easy to visualise and understand. The students get also pedagogically reasonable complex representation of system, since two translations and rotational phenomena exists. To describe the system completely a 3D pre-processor is needed. This is a task for future development, which is discussed in next chapter.

5.3 Future plans for use in education and research

This paper is solely focused on the development of the device and software infrastructure for vibration teaching. There exists three possible directions for future utilization of the developed device:

- Testing the pedagogical impact of demo device's impact on vibration education on basic courses. The evaluation should be parallel measuring the impact of traditional texts, the use of demo device and use of the developed Multi Body Program of this paper. The related properties to be evaluated should be at least [6]: Pre and post evaluation of conceptual skills, and subjective response of utilization of each software/device/text: enjoyment, usefulness, logical structurization, ease of use, reasonable time use, versatility to problems, equation understandability and ability for equation memorize.
- Testing the device in student project works in a learning-by-doing manner: Advanced project work for developing a piezo-electric accelerometer instrumentation: instrumentation, data acquisition, identification of vibration parameters to multi body program, verification of multi body program response to vibration device data acquisition results, and error analysis. Development of a 3D version of the vibration pre-processor as a student project.
- Research purposes: Use of the device and software for development of advanced systematic data acquisition algorithms, data mining of vibration to identify the system's parameters.

6. References

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